Exercise 12

Find dy/dx by implicit differentiation.

$$\cos(xy) = 1 + \sin y$$

Solution

Differentiate both sides with respect to x.

$$\frac{d}{dx}[\cos(xy)] = \frac{d}{dx}(1 + \sin y)$$

$$[-\sin(xy)] \cdot \frac{d}{dx}(xy) = \frac{d}{dx}(1) + \frac{d}{dx}(\sin y)$$

$$[-\sin(xy)] \cdot \left\{ \left[\frac{d}{dx}(x) \right] y + x \left[\frac{d}{dx}(y) \right] \right\} = (0) + (\cos y) \cdot \frac{d}{dx}(y)$$

$$[-\sin(xy)] \cdot \big[(1)y + x(y')\big] = (\cos y)(y')$$

$$-\sin(xy)(y + xy') = y' \cos y$$

Solve for y'.

$$-y\sin(xy) - xy'\sin(xy) = y'\cos y$$

$$-y\sin(xy) = [x\sin(xy) + \cos y]y'$$

$$y' = -\frac{\sin(xy)}{x\sin(xy) + \cos y}$$